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Rational bubbles in the real housing stock market: Empirical evidence from Santiago de Chile[☆]

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ABSTRACT

In this paper we study the presence of rational bubbles in the IGP and IAR real housing stock indexes of Chile during the period 2003:01 to 2016:03 using a methodology based on fractionally integrated and cointegrated processes. Our findings suggest strong evidence in favour of bubbles in the Chilean housing stock market when no breaks are taken into account. Testing for structural breaks, three break dates are detected at 2007, 2011 and 2014, and the same evidence in favour of bubbles holds. This can be explained by the high level of debt of the Chilean people.

1. Introduction

From 2003:01 to 2016:03 the Chilean economy experimented an extraordinary increase in housing prices of approximately 67%. Indeed, Santiago Housing Real Price Index (IGP), soared from 101.13 in January 2003 to 168.4 in March 2016. Still more remarkable is, during this time, the Housing Rent Price Index (IAR), used to measure the housing rent price variation in Santiago, also increased, but only from 99.2 to 112.3, an increment of 13% [1]. The differences between the two indexes and the persistent rise in housing stock prices over the period under analysis has led observers to ask whether the increase in real house prices responds to the evolution of structural economic variables, or if this is due to the existence of a bubble in the Santiago housing stock market. In this work, we test the presence of a rational bubble in the housing market in Santiago de Chile using a fractional integration and cointegration methodology for the price-rent ratio.

The term rational housing bubble indicates a possible deviation of the housing stock price from its fundamental value (Stiglitz, 1990). If fundamental variables are able to explain the increments in housing prices, a financial crisis is unlikely to take place considering that house prices will not fall unless fundamental variables change. On the contrary, if speculative variables are the reason behind the increase in prices, a rational housing bubble may be present causing various negative effects in the real economy (Stiglitz, 2003; Kivedal, 2013). In the Chilean case, for example, even when the bubble does not burst, the increase in prices might make it difficult for house seekers to buy a house. Indeed, housing sales in Santiago decreased approximately 39% in 2016.

On the other hand, if the bubble bursts, there is a possibility of a financial crisis. This could lead significant negative wealth effects

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[1] Details of the data are given in Section 4.

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for the families owning a house. In fact, Reinhart and Rogoff (2008) points out that after a financial crisis the real stock of debt nearly doubled. In Chile, for instance, household debt represents 63.2% of disposable income.

This high level of debt of Chileans and wage dispossession results in the Household Income ratio (HIR) being the highest in the OECD, with an average of 38%, just followed by the Netherlands with a significant 18.1%. A HIR of 38% means that Chilean people can only live with an approximately 62% of their real income. Thus, we believe that, in order to implement an optimal policy for Chile to deal with rational housing bubbles, a problem for policy makers is their detection.

The increment in houses prices is one of the most important features determining the existence of a rational housing bubble. However it is not conclusive evidence (Case and Shiller, 2003). The rental price, on the other hand, is also a fundamental variable to explain variations in house prices. As a result, analyzing the ratio between house price and the rental price could be a very good methodology for detecting rational housing bubbles (Himmelberg et al., 2005). In particular, for the Chilean housing stock market the real house price and the real rental price are measured by the IGP and IAR indexes respectively. These real indexes should move together in the long run since the opportunity cost buying a house is renting one. Therefore, if the increase in the IGP is not followed by a similar increase in the IAR, other variables different to the IAR must be present in order to explain large increases in housing prices. These variables could be represented by psychological factors causing a rational housing bubble.

Additionally, it is possible to view a house as an investment. In this case, the rentability from the house will then be the rental price which is received by the investor when he rents the house. Consequently, the relationship between the IGP and the IAR will be the same as the relationship between asset prices and dividends in a financial market. Moreover, the fundamental value of a house will be the present value of all the future cash flows of the house, such that a potential difference of the present price of the house from its fundamental value is a rational housing bubble Brooks et al. (2001).

Rational housing bubbles take place when the prices of the houses increase due to investors holding the belief that they will be able to sell the overvalued house at a higher price in the future Flood and Hodrick (1990). In general, models of rational bubbles imply that the relationship between asset prices (house prices) and their dividends (rent prices) can be used to investigate the presence of bubbles in all stock markets. There exists many authors who have used this relationship to analyse the existence of this economic problem (Campbell and Shiller, 1987, 1988; Diba and Grossman, 1988; Froot and Obstfeld, 1989; Caporale and Gil-Alana, 2002; Craine, 1993; Cuñado et al., 2005; Nunes and Da Silva, 2007; Gürkaynak, 2008; Kivedal, 2013; Miao, 2014; Engsted et al., 2016; Tran, 2016; Kim and Lim, 2016; Zhang et al., 2017; Astill et al., 2017, 2018; Harvey et al., 2018 among others). Most of these papers test the existence of housing and stock market bubbles using traditional unit-root tests to the price-dividend ratio. In this work, we represent the price-dividend ratio by the IGP-IAR ratio.

Campbell and Shiller (1987) used annual data for the S&P 500 from 1871 to 1986 to test the presence of rational bubbles. These authors found persistent deviations of stock prices using the present-value model. They also found out rational bubbles when co-integration test were applied between stock prices and dividends. Froot and Obstfeld (1989) and Craine (1993) also test for a unit root in the price-dividend ratio using annual data of the S&P 500 for 1900–1988 and 1876-1988 and obtained the same conclusion respectively. Diba and Grossman (1988), however, analyzing data from the S&P 500 index for 1871–1986, found that stock prices do not contain explosive rational bubbles. Tran (2016) tests for the existence of periodically collapsing stock price bubbles in Asian and Latin American emerging stock markets for the period 1990–2009. Using monthly data of price indexes and dividends, he found that the hypothesis of formation of bubbles cannot be rejected for all of the studied emerging stock markets

Caporale and Gil-Alana (2002), use a more flexible methodology for testing the null hypothesis of no cointegration against alternatives which are fractionally cointegrated in order to validate Present Value models of stock prices. They found out that stock prices and dividends are both I(1) nonstationary series, but they are fractionally cointegrated. In the same line of analysis, Cuñado et al. (2005), break with the tradition of using the S&P 500 index and the classic check of cointegration through fractionally integrated methods. They cannot reject the hypothesis of a rational bubble in the dividend-price ratio of shares from NASDAQ considering monthly series for the time period $1993 - 2004$.

On the other hand, many authors have tested for rational bubbles in real state markets. Kivedal (2013) investigated rational bubbles in the US housing market prior to the 2007 sub-prime mortgage financial crisis. Using the price-rent ratio, he suggests that there was a bubble in the housing market prior to the financial crisis, even when controlling for the fundamental information given by the rental price in the period and the decreasing interest rate. Miao (2014) also tests for rational bubbles using the price -rent ratio and found that fundamental factors alone cannot explain the housing price dynamics. However, Zhang et al. (2017) did not find evidence of rational bubbles. This author used a unique data-set of city-level house prices and rents in major Chinese city housing markets and demonstrated that the price-rent ratios in most of the sample cities are no longer non-stationary. Finally, Kim and Lim (2016) uses the present value model to analyze the dynamics of the Korean housing market. Using the price-rent ratio, he found that the bubble has continuously being accumulating since the early 2000*s*, reaching as high as 51% of the house price at the end of 2014.

Additionally, the literature that tests the presence of bubbles in the Chilean housing market is scarce. Bergoeing et al. (1999) test the presence of cointegration equations between prices and fundamentals to determine deviations from long-term prices. They found that there are periods where a deviation of asset prices from their fundamental value is very considerable, and therefore indicating the presence of a rational bubble. Parrado et al. (2009) study the evolution of housing prices, using two different sources of information and various methods of analysis. The results show that changes in economic fundamentals could largely explain the increase in prices. Finally, Idrovo and Lennon (2013) attempt to verify the existence of ”persistent shocks” or ”housing rational bubble” in the real price index of new housing (IGP) of Greater Santiago during the period January 1994 to October 2012. They use an error correction model to analyse the cointegration between the price index and its fundamentals and a model that estimates the importance of both fundamentals and the speculative component in the price of housing. They concluded that if speculative behaviour in the housing stock market exists, it would not be generalized among economic agents.

At this point we have to take into account the criticism of integration/cointegration based tests of Evans (1991) and Gürkaynak (2008) for testing bubbles. According to these authors, for each paper that finds evidence of bubbles, there is another one that fits the data equally well without allowing for a bubble. Moreover, the presence of time varying parameters or regime-switching fundamentals can produce biases in the results. In this sense, the use of fractional integration can partially solve this problem. Firstly, because we departure from the rigidity of integers degrees of differentiation ($d = 0$ in case of stationarity, and $d = 1$ for non-stationarity), and secondly, because fractional integration is very much related with regime-switching models and breaks (Diebold and Inoue, 2001; Granger and Hyung, 2004; etc.).

Some recent developments in the bubble-detection literature include, for example, sequential (recursive) testing procedures (see Phillips et al., 2011, 2015). These sequential tests have been designed to enable policy regulators to date-stamp the emergence of speculative bubbles, and offers a potential tool for real-time detection of stock market bubbles. Extensions of these methods have been elaborated in Astill et al. (2017, 2018), Chen et al. (2017), Harvey et al. (2018) and others. However, these methods do not depart from the strong I(0)/I(1) dichotomy and do not take into account potential fractional alternatives as is the case in this work.

In line with Caporale and Gil-Alana (2002) and Cuñado et al. (2005), we also test for rational bubbles using fractional integration and cointegration. In particular, the IGP-IAR ratio of the housing market of Chile will be used to analyze the existence of rational housing price bubbles. Therefore, the need for fundamentals variables to analyze the relative importance of each factor is eliminated. Moreover, instead of using the classic approach based on I(1)/I(0) integration and/or cointegration, we use fractional methods (Granger, 1980; Granger and Joyeux, 1980). The difference between IGP and IAR indexes will be analyzed. In particular, instead of the typical dichotomy type I(0)/I(1), a wide range of I(d) models is considered, where d is not necessarily restricted to 0 or 1. To this end, different methods of fractional integration will be employed, among them, a version of Robinson's (1994) parametric tests along with other semiparametric methods.

According to our results, we have found strong evidence of the existence of a rational bubble in the Chilean housing stock market. The main difference between this work and others that consider tests of rational bubbles in the housing market is that instead of using the traditional integration and cointegration methods developed among others by Dickey and Fuller (1979) (ADF tests for unit roots), and Engle and Granger (1987) and Johansen (1991, 1995a,b) (for cointegration), we use fractionally integrated approaches. To the best of our knowledge, there are no papers in Latin America for testing the presence of rational bubbles using fractional integration and cointegration methods.

The paper is organized as follows. The following section provides a brief discussion of the present value model under rational expectations, and introduces the notion of rational bubbles in housing stock markets. Section 3 presents the I(d) techniques in the analysis of rational bubbles in the Chilean housing market. In Section 4, the corresponding data used will be presented along with a description of the Housing Real Price Index (IGP), Nominal Rent index (IAN) and Real Rent Index (IAR), covering the period 2003:01–2016:03. Then, in Sections 5 and 6 the fractional integration test for rational bubbles with and without structural breaks will be preformed. Finally, Section 7 summarizes the empirical results and also contains some concluding comments.

2. Rational bubbles in housing stock markets

As pointed out before, in this section we discuss the present value model and the notion of rational bubbles in the context of housing stock markets.

According to Campbell et al. (1997) the return on a stock can be defined as

$$R_{t+1} = \left[\frac{P_{t+1} - P_t + D_{t+1}}{P_t}\right]. \tag{1}$$

where R_{t+1} can be regarded as the return of a housing stock held from time t to $t + 1$, P_{t+1} represents the housing price also held from t to $t + 1$ and it will be measured by the IGP index, while D_{t+1} is defined as the rent price (dividend in Eq. (1)) and will be quantified by the IAR index. The subscript $t + 1$ denotes the fact that the return only becomes known in period $t + 1$. In terms of the indexes, Eq. (1) can be written as

$$R_{t+1} = \left[\frac{IGP_{t+1} - IGP_t + IAR_{t+1}}{IGP_t}\right]. \tag{2}$$

If we apply the expected value over Eq. (2), considering all the information available at time t, the following equation is obtained

$$IGP_t = \mathbb{E}_t\left[\frac{IGP_{t+1} + IAR_{t+1}}{1 + R_{t+1}}\right]. \tag{3}$$

In order to obtain the semi-reduced form, we solve forward equation (2) k periods

$$IGP_t = \mathbb{E}_t\left[\sum_{t=1}^{k}\left(\frac{1}{1 + R_{t+1}}\right)^i IAR_{t+i}\right] + \mathbb{E}_t\left[\left(\frac{1}{1 + R_{t+k}}\right)^k IGP_{t+k}\right]. \tag{4}$$

To solve uniquely Eq. (4), it is necessary to assume that in the indefinite future, the expected discounted value of the housing stock converges to zero, i.e.,

$$\lim_{k\to\infty} \mathbb{E}_t\left[\left(\frac{1}{1+\mathrm{R}_{t+k}}\right)^k \mathrm{IGP}_{t+k}\right] = 0. \tag{5}$$

Eq. (5) is used to obtain the fundamental value of the housing stock (Z_t) as the sum of the expected discounted price rents (IAR_{t+i}) as follow:

$$Z_t = \mathbb{E}_t\left[\sum_{t=1}^{\infty}\left(\frac{1}{1+\mathrm{R}_{t+1}}\right)^i \mathrm{IAR}_{t+i}\right]. \tag{6}$$

However, if the convergence assumption in Eq. (5) is discarded, we get an infinite number of solutions, any one of which take the following form

$$\mathrm{IGP}_t = Z_t + \Psi_t. \tag{7}$$

From Eq. (7), Ψ_t can be expressed as:

$$\Psi_t = \mathbb{E}_t\left[\frac{\Psi_{t+1}}{1+\mathrm{R}_{t+1}}\right]. \tag{8}$$

In Eq. (8), the term Ψ_t is called a “rational bubble” and appears in housing prices only because it is expected to be present in the next period with a value $\Psi_t\mathbb{E}_t(1+\mathrm{R}_{t+1})$, and, in addition, it is entirely consistent with rational expectations and the time path of expected returns.

Since the presence of time-varying expected housing stock returns has led to a non-linear relation between prices and returns, we use a log-linear approximation to Eq. (2) suggested by Campbell and Shiller (1988). Therefore

$$r_{t+1} = \log(\mathrm{IGP}_{t+1} + \mathrm{IAR}_{t+1}) - \log(\mathrm{IGP}_t). \tag{9}$$

From Eq. (9), we get:

$$r_{t+1} = \mathrm{igp}_{t+1} - \mathrm{igp}_t + \log[1 + \exp(\mathrm{iar}_{t+1} + \mathrm{igp}_{t+1})]. \tag{10}$$

In Eq. (10), $r_{t+1} = \log(1+\mathrm{R}_{t+1})$, $\mathrm{igp}_t = \ln(\mathrm{IGP}_t)$, $\mathrm{igp}_{t+1} = \ln(\mathrm{IGP}_{t+1})$, $\mathrm{iar}_t = \ln(\mathrm{IAR}_t)$ and $\mathrm{iar}_{t+1} = \ln(\mathrm{IAR}_{t+1})$. Moreover, Eq. (10) can be regarded as a non-linear function of the log rent-price ratio or, more specifically, $\log(\mathrm{IAR}/\mathrm{IGP}) = (\mathrm{iar} - \mathrm{igp})$. If we approximate the last equation around the mean by a first-order Taylor expansion, the following equation is obtained

$$r_{t+1} \approx \theta + \psi \mathrm{igp}_{t+1} + (1-\psi)\mathrm{iar}_{t+1} - \mathrm{igp}_t. \tag{11}$$

In Eq. (11), θ and ψ are scalar parameters. Therefore, Eq. (11) is a linear difference equation for the log of housing stock price. After solving forward and imposing the no transversality condition, is possible to obtain

$$\mathrm{IGP}_t = \frac{\theta}{1-\psi} + \sum_{j=0}^{\infty}[(1-\psi)\mathrm{iar}_{t+1+j} - r_{t+1+j}]. \tag{12}$$

Taking mathematical expectation on Eq. (12) and based on information available at time t, we obtain

$$(\mathrm{iar}_t - \mathrm{igp}_t) = -\frac{\theta}{1-\psi} + \mathbb{E}_t\left[\sum_{j=0}^{\infty}\psi^j[(1-\theta)\mathrm{iar}_{t+1+j} - r_{t+1+j}]\right]. \tag{13}$$

In Eq. (13), if housing stock prices (igp_t) and real rents (iar_t) follow integrated processes of order one (no bubbles condition), the log housing stock price and the log rents are cointegrated with cointegrating vector (1, 1) and the log rent-price ratio ($\mathrm{iar}_t - \mathrm{igp}_t$), under no rational bubble restriction, is a stationary I(0) process. On the other hand, the presence of a unit-root or I(1) process in the log rent-price ratio is consistent with rational bubbles in housing stock markets.

In this context, the usual form of testing the absence of rational bubbles assumes that the individual series (log of real housing stock prices and log of real rents) are both non-stationary I(1) while their linear combination is I(0). However, instead of using classic methods, based on autoregressive (AR) models, we use fractionally integrated approaches which allow these orders of integration be potentially fractional. This will be discussed in the next section.

3. Methodology

As mentioned earlier we use I(d) techniques in the analysis of rational bubbles in the Chilean housing market. This approach is more general than the standard methods based on unit roots that simply consider the two cases of stationarity I(0) and non-stationarity I(1). By contrast, we allow d to be a real value, and thus, it may be a fractional one, constrained between 0 and 1, or even above 1. In other words, we say that a process $\{x_t, t = 0, \pm 1, \ldots\}$ is integrated of order d, and denoted as $x_t \sim I(d)$ if it can be represented as

$$(1-\mathrm{L})^d x_t = u_t, \quad t = 0, \pm 1, \ldots, \tag{14}$$

with $x_t = 0$ for $t \le 0$, and where L denotes the lag operator, i.e., $\mathrm{L}x_t = x_{t-1}$, and u_t is a covariance stationary I(0) process. In case of a fractional value for d, we can expand the polynomial in the left hand side in Eq. (14) as

$$(1-L)^d = \sum_{j=0}^{\infty} \psi_j L^j = \sum_{j=0}^{\infty} \binom{d}{j}(-1)^j L^j = 1 - dL + \frac{d(d-1)}{2}L^2 - ...,$$

such that Eq. (14) can be expressed as

$$x_t = dx_{t-1} - \frac{d(d-1)}{2}x_{t-2} + ...+u_t.$$

Thus, the differencing parameter becomes an indicator of the degree of persistence of the data, and higher the value of d is, higher the level of association between the observations is.

This specification allows for a large variety of models, including the classical I(0) or short memory processes (such as the white noise or the ARMA-type classes, in case of $d = 0$); long memory if $d > 0$, and within this latter class, we can distinguish stationary long memory ($0 < d < 0.5$) from nonstationarity ($d \geq 0.5$); finally, mean reversion takes places as long as d is smaller than 1, while lack of it takes places in the unit root or I(1) case or if d is above 1.

In the empirical application carried out in the following section, we consider the following regression model,

$$y_t = \alpha + \beta t + x_t \quad t = 1, 2, ..., \tag{15}$$

where y_t is the series under examination; α and β are unknown coefficients referring respectively to an intercept and a linear time trend, and the regression errors, x_t, are of the same form as in Eq. (14), that is, integrated of order d, where d is also an unknown parameter to be estimated. In the following section we test the null hypothesis

$$H_0: d = d_0 \tag{16}$$

for any real value d_0, in a model given by Eqs. (14) and (15). For this purpose we use a version of the tests of Robinson (1994). It is a Lagrange Multiplier (LM) test that is based on the Whittle function in the frequency domain (Dahlhaus, 1989). This test has a standard null limit distribution unlike what happens in most unit root methods where the critical values have to be computed numerically on a case by case simulation study. Moreover, this standard limit behaviour is unaffected by the presence of deterministic terms as those given in (16). The functional form of the test statistic can be found in any of the numerous empirical applications carried out with them (see, e.g. Gil-Alana and Robinson, 1997; Gil-Alana, 2000; Gil-Alana and Moreno, 2012; etc.). Along with this method, we also display estimates of the differencing parameter d, based on semiparametric approaches (Robinson, 1995; Abadir et al., 2007). Finally, based on the potential presence of breaks in the data, we also use Bai and Perron (2003) method of detecting multiple breaks in the data along with another approach that allows us to estimate the differencing parameters and thus the degree of persistence in the data, allowing for breaks, where the break dates as in Bai and Perron (2003), are endogenously determined by the data. (See, Gil-Alana, 2008).

4. The data

Fig. 1 displays the time series plots.

We used data corresponding to Housing Real Price Index (IGP), Nominal Rent index (IAN) and Real Rent Index (IAR), all in the original and log-transformed data, monthly, for the time period January 2003 - April 2016. The Real Housing Price Index (IGP), which constructs and publishes the Camara Chilena de la Construccion, considers the sales prices of new houses in the Housing Market of Santiago of Chile. The prices are measured in ”Unidad de Fomento” (UF), which is a daily unit of measure that is indexed to the evolution of the Chilean Consumer Price Index (IPC) of the previous month. The IPC is constructed by the Instituto Nacional de Estadisticas (INE) de Chile. As can be seen in the bottom left of Fig. 1, the IGP shows a growth of 68.4% throughout the period under study. In addition, this index remains relatively stable for much of the period 2003–2006, however, from 2007 until 2009 the IGP reveals a fairly systematic growth, at an average rate of 4.2% per month. Finally, from 2010 to 2016, there is a notable acceleration in the speed of growth of these prices reaching a monthly average of 6.8%.

The Nominal Rent Index (IAN), on the other hand, is a component of the Consumer Price Index (IPC). This proxy corresponds to the value of the housing rent actually paid in all the real estate market (new and second hand houses) in the main cities of the country, including Santiago. The housing rent are normally traded in Chilean pesos, that is, in nominal terms. The top left of Fig. 1 shows that the IAN increased by 73.5% from January 2003 to March 2016, representing a 19.4% higher increase than the 54.1% rise in the IPC during the same period. Furthermore, three sub-periods can be distinguished in the evolution of this index. First of these, from 2003 to 2006, there was a small increment (2.6%) in contrast to the IGP which remained stable. However, in the next sub-period (2007–2009) the IAN showed a remarkable increase of approximately 20.2% (almost a 16.1% higher than the IPC). In the last sub-period (2010–2016) the increment is more regular and systematic as the IAN increased by approximately 27.4%, which represents a remarkable 16.7% above the IPC.

Finally, the Real Rent Index (IAR) is obtained by deflating the IAN using the IPC in every month during the entire period under consideration. As can be seen on the middle left of Fig. 1, during all the period the IAR increased by only 12.6% (60.9% lower than the IAN for the same period). When the first sub-period (2003–2006) is considered, the IAR shows a decrease of 3%. However, from 2007 to 2009 the IAR showed a notable increase of 12%. Finally, when we consider the sub-period, 2010 − 2016, the growth of the IAR remained stable, registering a growth of approximately 5%.

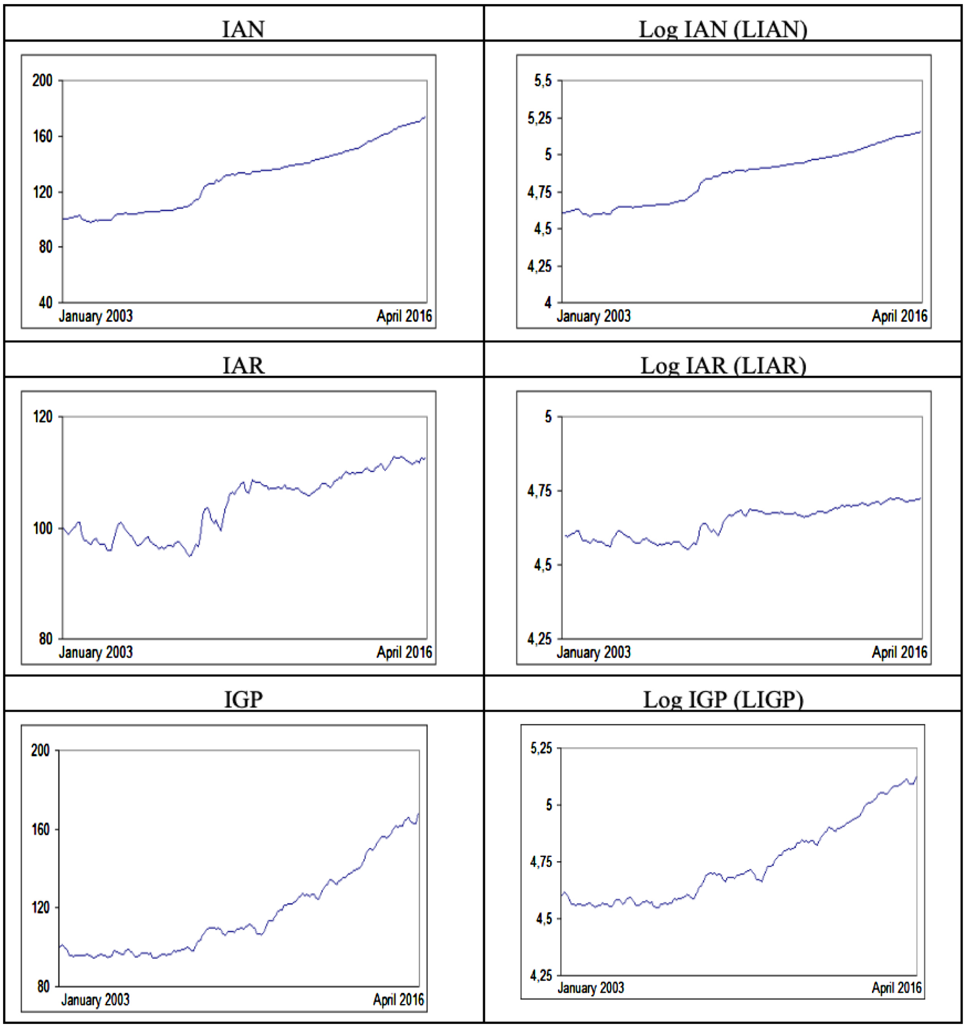

Fig. 1. Time series plots for the original and log-transformed data (January 2003 - April 2016).

5. Testing rational bubbles

We start this empirical section by examining the three series of prices in the Chilean housing market, and using the model given by Eqs. (14) and (15), testing H_0 (Eq. (16)) for the two cases of uncorrelated (white noise) and autocorrelated errors. In the latter case we use a non-parametric approach proposed by Bloomfield (1973) that approximates highly parameterized ARMA processes with a reduced number of parameters and which fits extremely well in the context of fractional integration. Table 1 focusses on the original data while Table 2 displays the results for the log-transformed data, and in all cases we present the results for the three cases of i) no deterministic terms ($\alpha = \beta = 0$), ii) with an intercept (α unknown and $\beta = 0$) and iii) with an intercept and a linear time trend (α and β unknown).

Tables 1 and 2 display the estimates of d along with the 95% confidence bands of the non-rejection values of d_0 using Robinson (1994) parametric test. In other words, we test H_0 (Eq. (16)) in Eqs. (14) and (15) for d_0-values equal to 0, 0.01, …, 1.99, 2, and choose the range of values of d_0 where H_0 cannot be rejected at the 5% level. The first thing we observe in these two tables is that for the original data, an intercept seems to be sufficient to describe the deterministic terms; however, for the log-transformed data, a time trend seems to be required. This is obtained by looking at the t-values of the coefficients of the deterministic terms in the d_0-differenced model, noting that under the null hypothesis, Eqs. (14) and (15) can be written as

$$(1 - L)^{d_0} y_t = \alpha (1 - L)^{d_0} 1_t + \beta (1 - L)^{d_0} t + u_t,$$

and based on the fact that u_t is I(0) by construction, the t-values on α and β apply. Focussing now on the differencing parameter, we observe similar values in both tables though the results differ depending on the specification of the error term. Thus, under white noise errors, the estimated value of d is slightly above 1.3 for IAN and IAR, and close to 1.20 for IGP, and the unit root hypothesis ($d = 1$) is decisively rejected in favour of $d > 1$ in the three series.

Table 1
Estimates of d for the whole sample using a parametric approach. In bold the selected models according to the deterministic terms. In parenthesis the 95% band of non-rejection values of d.

Original Series	No Regresors	An Intercept	A Linear Time Trend
	(i) White Noise Disturbances		
IAN	0.98	***1.31***	1.32
	(0.88, 1.11)	***(1.20, 1.47)***	(1.22, 1.48)
IAR	0.97	***1.30***	1.30
	(0.87, 1.09)	***(1.11, 1.55)***	(1.11, 1.55)
IGP	1.00	***1.18***	1.19
	(0.89, 1.15)	***(1.07, 1.37)***	(1.09, 1.38)
	(ii) Autocorrelated (Bloomfield) Disturbances		
IAN	0.98	***1.33***	1.33
	(0.88, 1.11)	***(1.21, 1.50)***	(1.22, 1.50)
IAR	0.98	***1.31***	1.31
	(0.88, 1.10)	***(1.12, 1.57)***	(1.12, 1.57)
IGP	0.98	1.18	***1.19***
	(0.88, 1.11)	(1.06, 1.37)	***(1.07, 1.38)***

Table 2
Estimates of d for the whole sample using a parametric approach. In bold the selected models according to the deterministic terms. In parenthesis the 95% band of non-rejection values of d.

Original Series	No Regresors	An Intercept	A Linear Time Trend
	(i) White Noise Disturbances		
IAN	0.98	0.98	***0.98***
	(0.88, 1.11)	(0.88, 1.11)	***(0.88, 1.11)***
IAR	0.97	0.97	***0.97***
	(0.87, 1.09)	(0.87, 1.09)	***(0.87, 1.09)***
IGP	0.97	0.97	***0.97***
	(0.87, 1.09)	(0.87, 1.09)	***(0.87, 1.09)***
	(ii) Autocorrelated (Bloomfield) Disturbances		
IAN	0.98	0.98	***0.98***
	(0.88, 1.11)	(0.88, 1.11)	***(0.88, 1.11)***
IAR	0.97	0.97	***0.97***
	(0.87, 1.09)	(0.87, 1.09)	***(0.87, 1.09)***
IGP	0.97	0.97	***0.97***
	(0.87, 1.09)	(0.87, 1.09)	***(0.87, 1.09)***

However, imposing autocorrelation throughout the model of Bloomfield (1973), the values are substantially smaller: 1.08 and 1.06 for IAN respectively for the original and log-transformed data; 0.77 and 0.73 for the IAR series, and 1.03 and 1.00 for IGP. In the cases of IAN and IGP the unit root cannot be rejected but this hypothesis is now rejected in favour of mean reversion (i.e., $d < 1$) in case of the IAR.

Due to the disparity in the results presented above depending on whether autocorrelation is assumed or not in the error term, in what follows we employ a semiparametric approach where no assumption is made on the structure of the error term. We use here a local 'Whittle' method (initially developed by Robinson (1995), and later extended and improved in Abadir et al. (2007). We display the results for a selected group of bandwidth number (m = 10, 11, …, 20) and the results show differences across the series (Tables 3 and 4). Thus, for IAN, the estimated value of d is found to be higher than 1, which is consistent with the results in Tables 1 and 2 with

Table 3
Estimates of d for the original data. In bold, evidence of mean reversion at the 5% level.

Bandwidth	d(IAN)	d(IAR)	d(IGP)	95% Lower I(1) Band	95% Upper I(1) Band
10	1.440	1.003	1.258	0.739	1.260
11	1.486	1.017	1.194	0.752	1.247
12	1.500	0.912	1.123	0.762	1.237
13	1.335	***0.731***	1.170	0.771	1.228
14	1.350	***0.729***	1.175	0.780	1.219
15	1.407	***0.755***	1.147	0.787	1.212
16	1.402	***0.746***	1.179	0.794	1.205
17	1.317	***0.739***	1.180	0.800	1.199
18	1.290	***0.754***	1.128	0.806	1.195
19	1.329	***0.776***	1.117	0.811	1.193
20	1.288	***0.793***	1.131	0.816	1.188

Table 4
Estimates of *d* for the logged data. In bold, evidence of mean reversion at the 5% level.

Bandwidth	d(IAN)	d(IAR)	d(IGP)	95% Lower I(1) Band	95% Upper I(1) Band
10	1.323	1.000	1.149	0.739	1.260
11	1.383	1.013	1.092	0.752	1.247
12	1.474	0.904	1.025	0.762	1.237
13	1.306	***0.729***	1.073	0.771	1.228
14	1.293	***0.723***	1.094	0.780	1.219
15	1.341	***0.749***	1.082	0.787	1.212
16	1.323	***0.742***	1.120	0.794	1.205
17	1.235	***0.735***	1.108	0.800	1.199
18	1.215	***0.750***	1.070	0.806	1.195
19	1.253	***0.772***	1.072	0.811	1.193
20	1.228	***0.790***	1.085	0.816	1.188

uncorrelated errors; for IAR, the estimates are generally below 1 and mean reversion is found in a number of cases, which is also consistent with the parametric results. Finally, for IGP most of the values are within the I(1) interval.

As a conclusion of these preliminary results we see that the three series, IAN, IAR and IGP (as well as their log-transformations) are nonstationary and highly persistent; however, while the orders of integration of IAN and IGP are found to be equal to or higher than 1, a small degree of mean reversion is detected in case of IAR, implying that shocks have a permanent nature in the former series but they are transitory and disappearing in the long run in the latter case.[2]

The follow-up step is to look at the possibility of bubbles by examining the order of integration in the ratio IGP / IAR as well as in its log-transformation. Table 5 displays the estimates again for the three standard cases of non deterministic terms, an intercept and a linear time trend. Focussing on the results based on autocorrelation, (which seems to be more realistic based on its more general formulation), we observe that the null hypothesis of I(1) behaviour cannot be rejected in any of the two series; moreover, a time trend is required in both cases. Thus, if no breaks are taken into account we observe strong evidence in favour of bubbles in the Chilean housing market.

6. Structural breaks

In this section we examine the presence of structural breaks in the data and perform the methodologies suggested in Bai and Perron (2003) and Gil-Alana (2008) for detecting breaks in nonstationary (and fractionally integrated) contexts.

The results are identical with the two procedures. Starting with the three individual series (IAN, IAR and IGP) the break dates almost coincide in the two cases of unlogged and logged data and they are displayed in Table 6. We observe 4 breaks in the case of IAN, with the breaks taking place at 2006M08, 2008M08, 2011M07 and 2013M09 with the original data, and slightly earlier (2006M01, 2008M03, 2011M02 and 2013M04) with the log-transformation, two breaks for the IAR series, both taking place at 2008M12 and 2013M04 and three breaks in the case of IGP at 2008M04, 2011M04 and 2014M04.

Next, and using now Gil-Alana (2008) methodology, we estimate the corresponding parameters for each series and each subsample, and the results are presented across Tables 7–12. Starting with the results for IAN (Table 6) we observe that no trend is required for the first two subsamples, but it is required in the last three subsamples. Moreover, the coefficient of the linear trend is increasing across the subsamples (from 0.32789 in the first subsample to 0.44481 in the second, and 0.73044 in the last one); on the other hand, dealing with the order of integration, we observe that the estimated value of *d* is found to be significantly above 1 in the first two cases, but *d* is in the I(1) interval in the last three subsamples. Nevertheless, and consistently with the previous results, mean reversion is not found in any single case for the IAN series. The results for IAR are rather similar, and an intercept is sufficient in the first two subsamples (ending at 2008M11 and at 2013M03) while a time trend is required in the last part of the sample. Moreover, the estimated value of *d* is found to be higher than 1 in the first subsample, while the I(1) hypothesis cannot be rejected in the second and third subsamples.

Focusing now on the log-transformed data (Tables 10–12), though quantitatively we observe some differences in the magnitude of the coefficients, qualitatively the same results hold: an intercept is sufficient for the log IAN series in the first two subsamples while a linear trend is required in the last three subsamples. The estimated value of *d* is now higher than 1 in the first subsample, but the I(1) hypothesis cannot be rejected in the rest of the cases. For the log of IAR, the time trend is only required in the last subsample, and the I(1) hypothesis cannot be rejected in the last two subsamples. For the log of IGP, the time trend is required in the last two subsamples as with the unlogged data. In any case, something that emerges from these results is the high level of persistence in the Chilean housing price structure, implying that a bubble is potentially present, with the effect of the shocks persisting forever. Fig. 2 displays the plots of the estimated trends for each case.

Next we test for breaks in IGP / IAR ratio and its log transformation, in order to check if the presence of bubbles detected on the whole sample could be a spurious phenomenon caused by the existence of breaks in the data. The results are displayed in Table 13.

[2] These results, however, might be biased due to the presence of breaks in the data. Thus, in the following section, the possibility of breaks is taken into account.

Table 5
Estimates of d for the whole sample using a parametric approach. In bold the selected models according to the deterministic terms. In parenthesis the 95% band of non-rejection values of *d*.

Original Series	No Regresors	An Intercept	A Linear Time Trend
	(i) White Noise Disturbances		
IGP / IAR	1.01	***1.20***	1.21
	(0.90, 1.15)	***(1.05, 1.45)***	(1.06, 1.45)
Log (IGP / IAR)	1.20	***1.21***	1.21
	(1.05, 1.43)	***(1.05, 1.44)***	(1.06, 1.44)
	(ii) Autocorrelated (Bloomfield) Disturbances		
IGP / IAR	0.88	0.96	***0.94***
	(0.72, 1.10)	(0.86, 1.12)	***(0.82, 1.13)***
Log (IGP / IAR)	0.93	0.93	***0.91***
	(0.83, 1.11)	(0.83, 1.11)	***(0.79, 1.12)***

Table 6
Break dates using Bai and Perron (2003) methodology for the IAN, IAR and IGP series.

Series	Number of Breaks	Break Dates
	(i) Original Series	
IAN	4	2006M08; 2008M08; 2011M07 and 2013M09
IAR	2	2008M12 and 2013M04
IGP	3	2008M04; 2011M04 and 2014M04
	(ii) Series in Log	
IAN	4	2006M01; 2008M03; 2011M02 and 2013M04
IAR	2	2008M12 and 2013M04
IGP	3	2008M04; 2011M04 and 2014M04

Table 7
Estimated coefficients for the IAN series. In parenthesis in the 3^{rd} and 4^{th} columns, their corresponding t-values.

IAN Series	*d* (95 % band)	Intercept (t-value)	Time trend (t-value)
1^{st} sub-sample	1.30	99.92606	...
	(0.03, 1.71)	(165.90)	
2^{nd} sub-sample	1.34	106.1105	...
	(0.08, 1.94)	(105.55)	
3^{th} sub-sample	0.95	127.639	0.32789
	(0.63, 1.53)	(233.12)	(4.20)
4^{th} sub-sample	0.80	139.2800	0.44481
	(0.20, 1.27)	(602.26)	(16.21)
5^{th} sub-sample	1.01	150.171	0.73044
	(0.79, 1.035)	(448.73)	(11.97)

Table 8
Estimated coefficients for the IAR series. In parenthesis in the 3^{rd} and 4^{th} columns, their corresponding t-values.

IAR Series	*d* (95 % band)	Intercept (t-value)	Time trend (t-value)
1^{st} sub-sample	1.33	100.2020	...
	(1.05, 1.77)	(124.50)	
2^{nd} sub-sample	1.08	103.5894	...
	(0.79, 1.42)	(200.41)	
3^{th} sub-sample	0.85	109.6677	0.07957
	(0.54, 1.34)	(302.82)	(2.11)

We observe the existence of three break points in the two series, corresponding to 2007M06 (2007M05 for the logged series), 2011M10 (2011M09) and 2014M04 and the estimated coefficients for the differencing parameters and the estimated trends are reported in Tables 14 (unlogged) and 15 (logged data).

We observe that the results are very similar in the two cases of unlogged and logged data. The first thing we know is that the confidence intervals are wide, which is not surprising based on the short sample sizes for the corresponding subsamples. The estimated values of *d* are 0.97 for the first subsample and 1.37 for the second one; and 1.01 (1.08 for the logged data) and 0.38 (0.42) respectively for the third and fourth subsamples. Thus, the largest estimate of d is obtained in the second subsample, while the lowest one in the fourth one. However, we also observe that for the first, third and fourth subsamples, we cannot reject the hypothesis of a

Table 9
Estimated coefficients for the IGP series. In parenthesis in the 3rd and 4th columns, their corresponding t-values.

IGP Series	*d* (95 % band)	Intercept (t-value)	Time trend (t-value)
1st sub-sample	***1.05***	100.0120	...
	(0.84, 1.38)	(101.76)	
2nd sub-sample	***1.36***	105.3386	...
	(1.07, 1.84)	(97.95)	
3th sub-sample	***0.85***	120.4175	0.06354
	(0.54, 1.27)	(107.56)	(5.36)
4th sub-sample	***0.47***	147.5879	0.08213
	(0.04, 2.17)	(128.17)	(9.83)

Table 10
Estimated coefficients for the LIAN series. In parenthesis in the 3rd and 4th columns, their corresponding t-values.

LIAN Series	*d* (95 % band)	Intercept (t-value)	Time trend (t-value)
1st sub-sample	1.31	4.60444	...
	(1.03, 1.77)	(714.12)	
2nd sub-sample	1.27	4.65527	...
	(0.79, 2.62)	(571.48)	
3th sub-sample	1.00	4.82143	0.00284
	(0.81, 1.31)	(1010.09)	(3.534)
4th sub-sample	0.94	4.92271	0.00285
	(0.68, 1.28)	(3154.78)	(11.27)
5th sub-sample	1.12	4.99966	0.00433
	(0.91, 1.44)	(2548.84)	(8.98)

Table 11
Estimated coefficients for the LIAR series. In parenthesis in the 3rd and 4th columns, their corresponding t-values.

LIAR Series	*d* (95 % band)	Intercept (t-value)	Time trend (t-value)
1st sub-sample	1.33	4.60720	...
	(1.05, 1.76)	(568.16)	
2nd sub-sample	1.08	4.64045	...
	(0.80, 1.42)	(960.82)	
3th sub-sample	0.85	4.69746	0.00071
	(0.54, 1.35)	(1442.85)	(2.11)

Table 12
Estimated coefficients for the LIGP series. In parenthesis in the 3rd and 4th columns, their corresponding t-values.

LIGP Series	*d* (95 % band)	Intercept (t-value)	Time trend (t-value)
1st sub-sample	***1.04***	4.60534	...
	(0.83, 1.38)	(457.93)	
2nd sub-sample	***1.35***	4.65734	...
	(1.06, 1.85)	(471.42)	
3th sub-sample	***0.83***	4.79156	0.00486
	(0.49, 1.26)	(562.07)	(5.63)
4th sub-sample	***0.55***	4.99424	0.00521
	(0.12, 2.59)	(649.50)	(8.71)

unit root, and this hypothesis is rejected in favour of d higher than 1 in the second one. Thus, we do not observe evidence of mean reversion in any of the four subsamples. Additionally, while an intercept is sufficient for the first two subsamples, a time trend is required in the last two, with the estimated trend coefficient being slightly higher in the last subsample.

According to these results the hypothesis of a bubble cannot be rejected since there is no evidence of mean reversion in any subsample and there is an increasing trend in the last two. However, we should also note that the last two subsamples are extremely short (containing 31 and 24 observations each). Therefore, as a final exercise, we have joined the last two subsamples in a single one in order to verify that the bubble exists in the Chilean housing market. The results in terms of the estimation of the differencing parameter and the time trend coefficients are displayed in Table 16.

We observe in this final table that though the confidence intervals are now smaller, the unit root hypothesis cannot be rejected in any of the two series for the extended subsample and once more the time trend coefficients are statistically significant in both cases, finding therefore support for the existence of a bubble in the Chilean housing market.

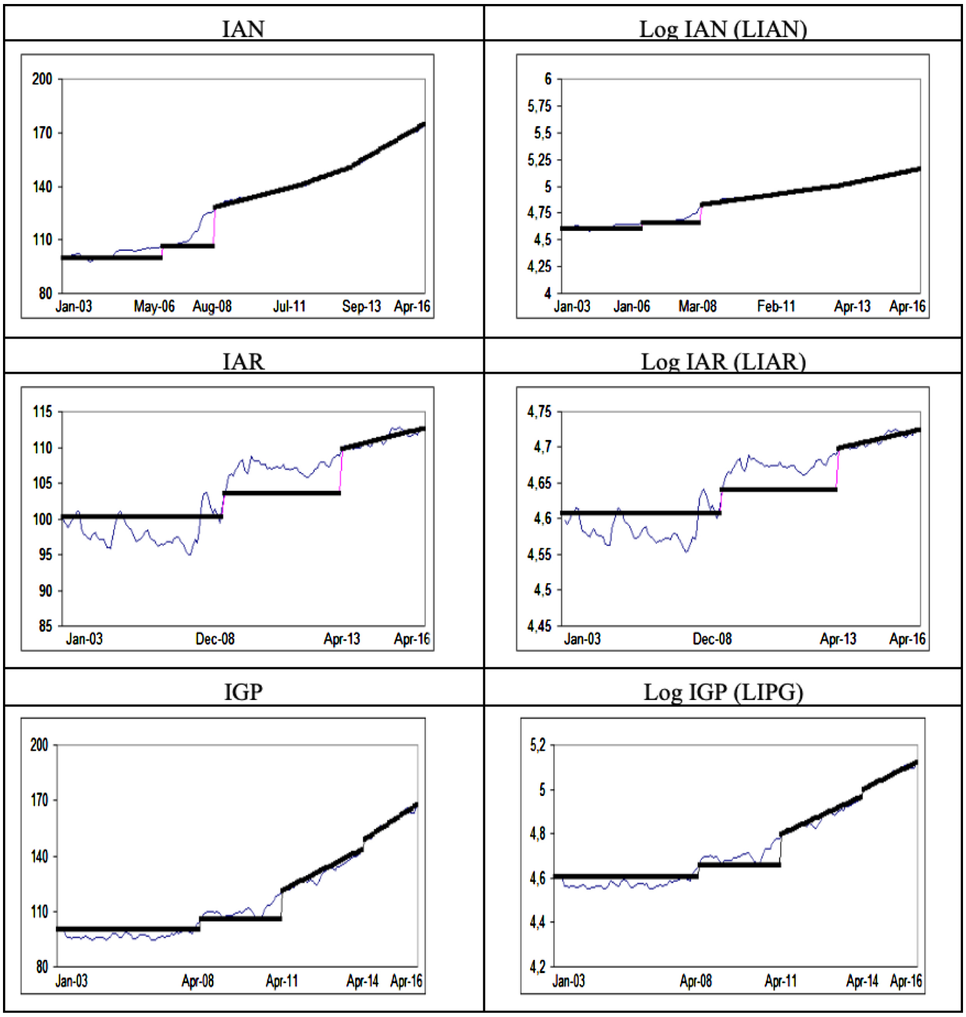

Fig. 2. Time series plots with estimated trends for the original and log-transformed data (January 2003 - April 2016).

Table 13
Break dates using Bai and Perron (2003) methodology for the IGP / IAR and Log (IGP / IAR) series.

Series	Number of Breaks	Break Dates
IGP / IAR	3	2007M06; 2011M10 and 2014M04
Log(IGP / IAR)	3	2007M05; 2011M09 and 2014M04

Table 14
Estimated coefficients for the IGP / IAR series. In parenthesis in the 3rd and 4th columns, their corresponding t-values.

IGP / IAR Series	*d* (95 % band)	Intercept (t-value)	Time trend (t-value)
1st sub-sample	0.97	1.00005	…
	(0.77, 1.20)	(90.73)	
2nd sub-sample	1.37	1.03597	…
	(1.12, 1.77)	(81.74)	
3th sub-sample	1.01	1.17475	0.00433
	(0.62, 1.57)	(94.19)	(1.84)
4th sub-sample	0.38	1.3376	0.00624
	(−0.66, 2.33)	(121.83)	(8.26)

Table 15
Estimated coefficients for the Log (IGP / IAR) series. In parenthesis in the 3rd and 4th columns, their corresponding t-values.

Log (IGP / IAR) Series	d (95 % band)	Intercept (t-value)	Time trend (t-value)
1st sub-sample	0.97	0.00008	...
	(0.65, 1.40)	(1.73)	
2nd sub-sample	1.37	0.01613	...
	(1.11, 1.77)	(1.91)	
3th sub-sample	1.08	0.13830	0.00429
	(0.54, 1.67)	(13.09)	(1.77)
4th sub-sample	0.42	0.29152	0.00442
	(−0.46, 2.35)	(36.33)	(7.85)

Table 16
Estimated coefficients for the two series (2014M04 – 2016M03). In parenthesis in the 3rd and 4th columns, their corresponding t-values.

Series	d (95 % band)	Intercept (t-value)	Time trend (t-value)
(IGP / IAR)	1.02	1.17309	0.00605
	(0.76, 1.65)	(80.39)	(2.83)
Log (IGP / IAR)	1.07	0.13790	0.00493
	(0.74, 1.71)	(12.42)	(2.55)

7. Summary and conclusions

In this paper we have examined the presence of rational bubbles in the real housing stock market in Santiago of Chile. The fractional integration and cointegration methodology was implemented using the IGP and IAR indexes which are used to measure the real price and real rent. We have also used these indexes as proxies to the price-rent ratio, i.e., IGP/IAR ratio, to implement more robust fractional integration / cointegration tests.

A rational bubble may bring serious consequences in the real economy of Chile even if the bubble does not burst. When this is the case, the increment of prices in the real housing stock market might imply that house seekers cannot buy a house. On the contrary, if the bubble bursts, a possible financial crisis could lead to large negative wealth effects for Chilean families who own a house.

In addition, after the formation of price bubbles in the real estate market, there are usually overly high levels of household debt (Reinhart and Rogoff, 2008). This is generally the case in contexts where there are contagions of expectations of excessive growth in the price of these assets that are often explained by excess liquidity in the economies. The latter induces unwanted behaviour in the supply of credit, and in the structure of interest rates, which encourages a greater demand for housing (Dominguez and Reinhart, 2008). Even when we consider that this situation could have arisen in Chile during the period under study, it could be the subject of future research.

After implementing a number of parametric and semiparametric methods, we found strong evidence supporting the existence of a rational bubble in the real stock housing market in Chile. This finding is obtained whether or not structural breaks are taken into account, and is consistent with other works that have also tested the presence of rational bubbles using data of other countries and different stock markets.

Finally, we firmly believe that this work can be extended in many directions. One possibility, for example, could be to include variables that can help to understand the fundamental value of the real price. In particular, the idea would be to incorporate the mortgage interest rate and real income along with the price-rent ratio to specify a multivariate time series model. Other possibility could be to conduct additional methods like the Generalized Supremum Augmented Dickey-Fuller (GSADF) test suggested in Phillips et al. (2011, 2015) and extended later by several authors (e.g., Astill et al., 2017, 2018; Chen et al., 2017; Harvey et al., 2018). Note, however, that the GSADF and its extensions are, in essence, a rolling window ADF style regression tests, and therefore, based on the strong I(0)/I(1) dichotomy and not considering potential fractional alternatives. With respect to the possibility of time varying parameters, we have partially solve this issue by means of allowing for structural breaks though other alternatives like more flexible nonlinear models still in the context of fractional integration (see, e.g. Cuestas and Gil-Alana, 2016) can be performed. These lines of research will be examined in future papers.

References

Abadir, K.M., Distaso, W., Giraitis, L., 2007. Nonstationarity-extended local whittle estimation? J. Econometr. 141 (2), 1353–1384.
Astill, S., Harvey, D.I., Leybourne, S.J., Sollis, R., Robert Taylor, A., 2018. Real-time monitoring for explosive financial bubbles? J. Time Ser. Anal. 39 (6), 863–891.
Astill, S., Harvey, D.I., Leybourne, S.J., Taylor, A.R., 2017. Tests for an end-of-sample bubble in financial time series? Econometr. Rev. 36 (6-9), 651–666.
Bai, J., Perron, P., 2003. Computation and analysis of multiple structural change models. J. Appl. Econometr. 18, 1–22.
Bergoeing, R., Morandé, F., Soto, R., et al., 1999. Asset Prices in Chile: Facts and Fads. ILADES.
Bloomfield, P., 1973. An exponential model for the spectrum of a scalar time series. Biometrica 60, 217–226.
Brooks, C., Katsaris, A., McGough, T., Tsolacos, S., 2001. Testing for bubbles in indirect property price cycles? J. Property Res. 18 (4), 341–356.
Campbell, J., Lo, A., Mackinlay, A.C., 1997. The Econometrics of Financial Markets. Princeton, Press.
Campbell, J.Y., Shiller, R.J., 1987. Cointegration and tests of present value models? J. Polit. Econ. 95 (5), 1062–1088.

Campbell, J.Y., Shiller, R.J., 1988. The dividend-price ratio and expectations of future dividends and discount factors? Rev. Finan. Stud. 1 (3), 195–228.
Caporale, G.M., Gil-Alana, L.A., 2002. Fractional integration and mean reversion in stock prices? Quart. Rev. Econ. Finan. 42 (3), 599–609.
Case, K.E., Shiller, R.J., 2003. Is there a bubble in the housing market? Brooking Papers Econ. Activity 2003 (2), 299–342.
Chen, Y., Phillips, P.C., Yu, J., 2017. Inference in continuous systems with mildly explosive regressors? J. Econometr. 201 (2), 400–416.
Craine, R., 1993. Rational bubbles: A test? J. Econ. Dynam. Control 17 (5-6), 829–846.
Cuestas, J.C., Gil-Alana, L.A., 2016. A non-linear approach with long range dependence based on chebyshev polynomials. Stud. Nonlinear Dynam. Econometr. 23, 445L 468.
Cuñado, J., Gil-Alana, L.A., De Gracia, F.P., 2005. A test for rational bubbles in the Nasdaq stock index: a fractionally integrated approach? J. Bank. Finan. 29 (10), 2633–2654.
Dahlhaus, R., 1989. Efficient parameter estimation for self-similar processes. Ann. Stat. 17, 1749–1766.
Diba, B.T., Grossman, H.I., 1988. Explosive rational bubbles in stock prices? Am. Econ. Rev. 78 (3), 520–530.
Dickey, D.A., Fuller, W.A., 1979. Distribution of the estimators for autoregressive time series with a unit root? J. Am. Stat. Assoc. 74 (366a), 427–431.
Diebold, F.X., Inoue, A., 2001. Long memory and regime switching? J. Econometr. 105 (1), 131–159.
Dominguez, K.M., Reinhart, C.M., 2008. Financial crash, commodity prices, and global imbalances. comments and discussion. Brooking Papers Econ. Activity 2008, 56L 68.
Engle, R.F., Granger, C.W., 1987. Co-integration and error correction: representation, estimation, and testing. Econometrica 55, 251–276.
Engsted, T., Hviid, S.J., Pedersen, T.Q., 2016. Explosive bubbles in house prices? Evidence from the OECD countries. J. Int. Finan. Mark. Inst. Money 40, 14–25.
Evans, G.W., 1991. Pitfalls in testing for explosive bubbles in asset prices? Am. Econ. Rev. 81 (4), 922–930.
Flood, R.P., Hodrick, R.J., 1990. On testing for speculative bubbles? J. Econ. Perspect. 4 (2), 85–101.
Froot, K.A., Obstfeld, M., 1989. Intrinsic Bubbles: The Case of Stock Prices. National Bureau of Economic Research Technical report.
Gil-Alana, L.A., 2000. Mean reversion in the real exchange rates? Econ. Lett. 69 (3), 285–288.
Gil-Alana, L.A., 2008. Fractional integration and structural breaks at unknown periods of time? J. Time Ser. Anal. 29 (1), 163–185.
Gil-Alana, L.A., Moreno, A., 2012. Uncovering the us term premium: an alternative route? J. Bank. Finan. 36 (4), 1181–1193.
Gil-Alana, L.A., Robinson, P.M., 1997. Testing of unit root and other nonstationary hypotheses in macroeconomic time series? J. Econometr. 80 (2), 241–268.
Granger, C.W., 1980. Long memory relationships and the aggregation of dynamic models? J. Econometr. 14 (2), 227–238.
Granger, C.W., Hyung, N., 2004. Occasional structural breaks and long memory with an application to the s&p 500 absolute stock returns? J. Empirical Finan. 11 (3), 399–421.
Granger, C.W., Joyeux, R., 1980. An introduction to long-memory time series models and fractional differencing? J. Time Ser. Anal. 1 (1), 15–29.
Gürkaynak, R.S., 2008. Econometric tests of asset price bubbles: taking stock? J. Econ. Sur. 22 (1), 166–186.
Harvey, D.I., Leybourne, S.J., Zu, Y., 2018. Testing explosive bubbles with time-varying volatility. Econometr. Rev. 1–21.
Himmelberg, C., Mayer, C., Sinai, T., 2005. Assessing high house prices: Bubbles, fundamentals and misperceptions. J. Econ. Perspect. 19 (4), 67–92.
Idrovo, B., Lennon, J., 2013. Methods for detection housing bubble: Evidence from Chile, MPRA paper, Munich Personal repec archive, number 44741.
Johansen, S., 1991. Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models. Econometrica 59 (6), 1551–1580.
Johansen, S., 1995a. Identifying restrictions of linear equations with applications to simultaneous equations and cointegration? J. Econometr. 69 (1), 111–132.
Johansen, S., 1995b. Likelihood-based Inference in Cointegrated Vector Autoregressive Models. Oxford University Press on Demand.
Kim, J.R., Lim, G., 2016. Fundamentals and rational bubbles in the Korean housing market: A modified present-value approach. Econ. Model. 59, 174L 181.
Kivedal, B.K., 2013. Testing for rational bubbles in the us housing market. J. Macroecon. 38, 369L 381.
Miao, J., 2014. Introduction to economic theory of bubbles. J. Math. Econ. 53, 130L 136.
Nunes, M., Da Silva, S., 2007. Rational bubbles in emerging stockmarkets, MPRA paper, Munich personal repec archive, number 4641.
Parrado, E., Cox, P., Fuenzalida, M., et al., 2009. Evolution of housing prices in chile? J. Econ. Chil. (The Chilean Economy) 12 (1), 51–68.
Phillips, P.C., Shi, S., Yu, J., 2015. Testing for multiple bubbles: Historical episodes of exuberance and collapse in the s&p 500? Int. Econ. Rev. 56 (4), 1043–1078.
Phillips, P.C., Wu, Y., Yu, J., 2011. Explosive behavior in the 1990 nasdaq: When did exuberance escalate asset values? Int. Econ. Rev. 52 (1), 201–226.
Reinhart, C.M., Rogoff, K.S., 2008. Is the 2007 us sub-prime financial crisis so different? An international historical comparison. Am. Econ. Rev. 98 (2), 339–344.
Robinson, P.M., 1994. Efficient tests of nonstationary hypotheses? J. Am. Stat. Assoc. 89 (428), 1420–1437.
Robinson, P.M., 1995. Gaussian semiparametric estimation of long range dependence. Ann. Stat. 23, 1048–1072 23.
Stiglitz, J.E., 1990. Symposium on bubbles? J. Econ. Perspect. 4 (2), 13–18.
Stiglitz, J.E., 2003. The Roaring Nineties. A New History of the World's Most Prosperous Decade. Norton, NY.
Tran, T.B.N., 2016. Speculative bubbles in emerging stock markets and macroeconomic factors: A new empirical evidence for Asia and Latin America. Res. Int. Bus. Finan (in press).
Zhang, D., Liu, Z., Fan, G.-Z., Horsewood, N., 2017. Price bubbles and policy interventions in the Chinese housing market. J. Hous. Built Environ. 32 (1), 135–155.