Información Importante

RISK MEASURES AND BID – ASK SPREADS OF OPTIONS OVER STANDARD AND POOR´S INDEX

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ABSTRACT

This paper aims to explain the bid-ask spread of options over the Standard and Poor´s Index on Chicago Board Options Exchange (CBOE) and how is explained by the Greek Letters of Options. Based on Maria E. de Boyrie, Yong O.Kim, Simon J. Park; the contribution of this paper is to relate and extend this theory from currency options to index options and demonstrate the significance of the Greeks on the options over the S&P Index

It was found that, except for Delta, all the Greek letters are significant explanatory variables of Bid Ask Spread on the CBOE options quoted over S&P's Index; specifically, Bid Ask Spread decreases with the convexity of the option (Gamma) and increases with the change in index volatility (Vega).

Keywords: *Greek Letters, Bid Ask Spread, simple regression model, index options.*

RESUMEN

Este documento tiene por objeto explicar el spread de las opciones sobre el índice Standard and Poors del Chicago Board Options Exchange (CBOE) y cómo se relacionan con las letras griegas. Con base en Maria E. de Boyrie, YongO.Kim, Simon J. Park, la contribución de este trabajo es relacionar y extender dicha teoría desde la perspectiva de opciones sobre divisas a opciones sobre índices y demostrar la significancia de las letras griegas sobre las principales opciones que integran el índice S&P.

Se encontró que, a excepción de Delta, las letras griegas son variables explicativas importantes del Bid Ask Spread sobre opciones CBOE cotizados sobre el índice S&P, en concreto, el Bid Ask Spread disminuye con la convexidad de la opción (Gamma) y aumenta con el cambio en el índice de volatilidad (Vega).

1. Introduction

Financial Markets have been evolving and more institutions have increasingly been vanguard on the structure of financial derivatives and its transactions. The Chicago Board Option Exchange (CBOE) promotes the negotiation of such instruments and is an important investment alternative to agents interested of mitigates systematic risk. Options are contracts that give the buyer the right, not the obligation, to buy or sell assets, securities, stock exchange indices, etc., at a specific price at a specific period of time. The options that give the right to buy receive the name of Calls and those that give the right to sell are called Puts. Options are highly influenced by factors such as volatility, time, price, interest rates, macroeconomic environment, and other factors that have been subjects of study by several authors [4].

One of the most important factors in analyzing the options is the Bid – Ask Spread that is a measure of liquidity which behavior depends on market movements. This spread represents a difference between purchase price and selling price and can be related to volatility, amount of the market and macroeconomic factors that can be reflected on option prices. The most common applications found using Greek letters over options are related to hedging strategies conforming specific portfolios. Some of them consider combined positions in some Call or Put options analyzing its effects (Greeks) independently. The effect of Greek letters on Bid – Ask spreads in both Call and Put options are been concentrated only using foreign exchange

Empirical evidence found in Bid – Ask spreads in USA also concern on Bond cases as done for Chakravaty and Sarkar in 1999 related to liquidity in U.S fixed income markets comparing corporate, government and municipal bond markets spreads, so, The contribution of this paper is to expand this analysis for the principal options in Standard and Poor´s Index listed in CBOE, finding how the parameters of the Greek letters affect its behavior.

2. Theoretical Framework

Bid is referred to the price at which the buyer is willing to buy, and Ask will be the price at which the seller is willing to sell. Considering the before statement, the lower the difference between parameters or Spread between Bid and Ask, the more liquid the market will be[1]. George and Longstaff (1993) argued that Call and Put Options differed only in Strike Price and expiration date and questioned how Spreads can be comparable between options.

An empirical study made by George and Longstaff (1993) conclude that almost 50% of negotiations are explained by the Spread variations and contracts related with maturity dates, specifically with closer maturity, have a bigger spread than options with more distant maturities. Most traders and researchers have focused on measure the three principal Bid Ask Spread cost namely inventory holding, adverse selection and other processing cost incurred by liquidity suppliers. Huang and Stoll (1997) quoted several statistical empirical models that measures the components of Bid Ask Spread pioneered by Roll (1984) who inferences that bid – ask spread is made from covariance properties of transaction prices. Years later, Glosten and Harris (1988) extend the study on a regression model but fails on the attainment of quoted data[2].

A recent study made by Boyrie, Kim and Park (2006) explained the bid and ask spread of currency options listed in Philadelphia Stock Exchange (PHLX) and the effects of price risk measures on its behavior. It was found that the bid – ask spread for currency options can be attributable to other price risk measures that are significant explanatory variables such as Greek Letters. They found that Delta and Gamma are the

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[1]Taken from http://www.mrtrader.com.ar/?p=188

[2] Please see Roger and Hans (1997) for more bid – ask spread empirical studies.

most significant variables and Bid Ask Spread increases with the change in option prices (Delta) and decreases with the convexity of the option (Gamma) [1]. In concrete this Paper aims to extend this theory to index options and demonstrate the significance of the other Greek Letters.

The most common applications found using Greek letters over options are related to hedging strategies conforming specific portfolios. Some of them consider combined positions in some Call or Put options analyzing its effects (Greeks) independently. The effect of Greek letters on Bid – Ask spreads in both Call and Put options are been concentrated only using foreign exchange asBoyrie, Kim and Pak (2006) did on currency options quoted on Philadelphia Stock Exchange (PHLX) analyzing the effects of Greek letters over a specific options

3. The Model

The measures expressed in Greek letters include: Delta, Gamma, Theta, Rho and Vega as its principals. In this concrete application, delta and gamma related to the sensibility of option price considering changes in the S&P index. Vega measures how the volatility in the index affects the price of options; rho will consider changes in the reference interest rate and Theta take into account option´s maturities. In concrete, the results tells that gamma takes a negative relation incrementing the options spread.

Each Greek letter measuresa different aspect of risk in a determined options position and the objective to the investor is managing them in order that all risks are acceptable. Most of traders look for hedging strategies using delta, gamma, theta, rho and Vega

The study focused onEuropean options over S&P index with maturity of January-2013 considering the short maturity date and the availability of data. Options are traded inEURUSD exchange rate that represents the mayor percentage of representativeness in the index. The equations followed in the regression and its expected signs are:

$$Bid - AskSpread_{call} = \beta_0 + \beta_{1\,Delta_{call}} + \beta_{2Gamma_{call}} + \beta_{3Vega_{call}} + \beta_{4Rho_{call}} + \beta_{5Theta_{call}} + \varepsilon$$

$$Bid - AskSpread_{Put} = \beta_0 + \beta_{1\,Delta_{put}} + \beta_{2Gamma_{put}} + \beta_{3Vega_{put}} + \beta_{4Rho_{put}} + \beta_{5Theta_{put}} + \varepsilon$$

Based on Hull(2009); Delta is the exchange rate of the option price to the underlying asset; in terms of the analysis of this paper, is the exchange rate of the spread in the option to the S&P´s Index where the slope of the curve relates the price of the option with the index. For European Call options, the delta is constructed as follows:

$$\Delta = N(d_1)$$

Where $\boldsymbol{d_1}$ is the Black – Sholes parameter for valuate options determined by

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

The formula provides the delta of a long position in the Call option.

Analogously for a Put option, the delta is equal to

$$\Delta = N(d_1) - 1$$

Notice that delta is positive for a long position in a Call option and negative for a long position in a Put option; in that sense, it is expected that delta has a positive relation with Call spread and negative for Put.

Gamma is the exchange rate of delta respect to movements in the value of the index. if gamma is small means that delta changes slowly and adjustments must to be made only at times. On the contrary, if gamma is too large means that delta is high sensitive to changes in index price. As Boyrie, Kim and Park (2006) states, Gamma

measures convexity of options so the higher (lower) the bid (ask) price the higher gamma option. In that sense, is expected that a larger gamma results in a lower bid – ask spread and it´s coefficients will be negative. In order to calculate Gamma, this parameter does not differentiate between Call and Put options, so for both, Gamma is calculated as follows

$$\Gamma = \frac{N'(d_1)}{(S_0)\sigma\sqrt{T}}$$

The nominator term is referred to the probability distribution for a standard normal distribution. It is a general result that can let conclude about the sign of this parameter; if a long position is considered, the gamma will be always positive and varies with *So* and options with very short maturities had a gamma very large. This conclusion will be crucial in the results obtained for call and put spreads.

On the other hand, Theta measures the change in the price of the option under changes in time, or, as time passes. For a European Call option,

$$\Theta(\text{call}) = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2)$$

And for an European Put option:

$$\Theta(\text{put}) = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT}N(-d_2)$$

Considering that Theta measures the time value loss, there is no accuracy on the effect of this parameter on bid – ask spread so it´s coefficients signs are ambiguous. Nonetheless the previous equations show that Theta is expected to be larger for Put than for Call.

The parameter Vega, measure the sensitivity of the option price considering a change in the volatility of the index. It is analyzed in absolute terms and the higher the

Vega the higher the sensitiveness of the option or portfolio under small changes in volatility. In that sense Vega is expected to be positive related to bid – ask spread, so the larger the Vega the larger the spread[3].It is calculated for Call and Put options equally following:

$$v = (S_0)\sqrt{T}N'(d_1)$$

Finally, Rho considers the effect of interest rates in the sensibility of option price. If it is considered an European Call option, the parameter is obtained by

$$rho = KT(e^{-rT})N(d_2)$$

And for European Put option

$$rho = -KT(e^{-rT})N(-d_2)$$

The price of call options increases with domestic interest rates and put option price decreases, so, it is expected that rho has a positive and larger effect on call options and negative for puts.In those terms, the signs obtained for these parameters also tell the direct sensibility in option spread under changes in interest rates in USA.

3.1 Methodology

Options information where obtained from data base registered in the web site of Chicago Board Options Exchange CBOE; there were found all options traded in the different indices. It was chosen as an Index of reference the Standard and Poor´s 500 index considering the availability of data and the purpose of this investigation over European options. All type of maturities could be selected and it was selected options that matures in January of 2013 because there were the closer liquid options listed in

[3]Additionally, Boyrie, Kim and Pak (2006) consider that in order to examine the effects of exchange rate volatility changes on option prices; it must be assume that the volatility is expected to remain constant over the life of the options.

the data base. There were available last trade prices, percent change, Bid, Ask, Volume, Delta, Gamma, Vega and Theta. After downloading data it was notice that Gamma was cero in all the cases so it was necessary to calculate them for both Call and Put Options. Also in the case of Rho that was not available. The spread was obtained by the subtraction *(Ask –Bid)*[4].

It was selected a representative sample of all options listed in the S&P´s index with maturity January of 2012. Approximately there were 250 options from which were taken 170 calls and 172 puts considering that in some of them there were no available data. There were no missing values in the data other than gamma that were corrected by constructed them by hand. The scheme of the data found in CBOE is in Table A.

By clicking each mnemonic is obtained the specific information for each option. For example, a Call Option has information shown in Table B.

This procedure where made for all 170/172 options selected constructing the adequate data base.

At the time of constructing both Gamma and Rho, was taken into account the current USA interest rate of 0,25%, spot price of the Standard and Poor´s index on Friday December7th of 2012 equal to 1418.07 and volatility in S&P´s index measure by the VIX registered a volatility of 16% in the same date.

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[4]A-priori knowledge.

4. Results

According to the theory exposed in the previous part of the paper, considering the signs of the coefficients, the results obtained for Call (Table 1) and Put (Table 2) are consistent. Despite of the statistical insignificance of Delta coefficient; as expected had a negative impact in bid – ask spread of Put Options and positive for Call´s meaning that a change in the price of the S&P´s index affects negatively (positively) the price of the Put (Call) option and consequently its spread.

Table I. Bid – Ask spread for Call Options[5]

	Coefficients		Standard Desviation
Delta	0.0639		0.0406
Gamma	-275.8132	***	10.4242
Vega	0.0007	***	0.0001
Theta	-6.3073	***	0.2697
LogRho	0.3297	***	0.0122
Coor	-0.2341	***	0.0270
Corr2	0.5234	***	0.0277
Corr3	-0.9381	***	0.1158
Constant	0.1814	***	0.0324
R^2	0.9699		
Obsevations	169		

[5]Corr, corr2 and corr3 are corrective errordummys created to satisfy normality conditions.

Table II: Bid – Ask Spread for Put Options

	Coefficients		Standard Desviation
Delta	-0.6589		0.5532
Gamma	-375.3156	***	42.0261
Vega	2.8940	***	0.3858
Theta	5.0493	***	1.2332
Rho	-0.0075	**	0.0036
Constant	0.1685	***	0.0228
R^2	0.9657		
Observations	171		

In the case f Gamma, Vega, Theta and Rho, all of them are statistical significant at or less than 0,5% for both equations. Gamma expected negativevalue sustains that a marginal change in Delta affects negatively the change in spread price of Put and Call options.Continuing with parameter Vega, the positive and small coefficient considers a soft sensibility of Call spread to changes in volatility. The contrary occurs with Put options where the effect of Vega parameter is higher. In both cases the parameter affects positively bid – ask spread as expected and is consistent with the theory, as Hull (2009) mention, for higher absolute values of Vega there is more sensitiveness to small changes in volatility and vice versa.

The negative sign of rho tells that Put option spread is sensitive (but not too much) to changes in interest rate in USA. The variable LogRho considered for Call options was created in order to minimize de high volatility, dispersion and kurtosis of the original parameter so it was necessary to take the variable in logarithms. The

interpretability considers that the bid – ask spread for Call Options change positively under variations on the logarithm of Rho.

Finally, bid – ask spread for Put options increase as time passes by and the option is closer to its maturity date. The contrary occurs for Call options where changes in Theta represent a negative effect on the bid – ask spread.

Post estimation parameters (Appendix) allows to conclude that both regressions are homoskedastic in which case Breush-Pagan/Cook – Weisberg test not reject the null hypothesis of homoscedasticity with a probability of 0,20 for Call and 0,06 for Put[6], meaning that other explanatory terms not explained by the model has no any relation with independent variables.

Ramsey test for Call options proves at a probability of 0,06 that the model has no omitted values and can not be rejected the null hypothesis. For Put regression the result was not statistically significant and may be due to the low homocedasticity of the errors that can be strengthened through error corrections as was done for Call regression.

Finally, null hypothesis of normality in the model can not be rejected with p-values above 0,5 in Shapiro – Francia normality tests for both Call and Put regressions. Above allows to conclude that the expected value of error term given the independent value is null and it behaves has a normal function of the residuals.

The results obtained support the theory proposed by John C. Hull 2009. In his book "Options, Futures and other Derivatives" consider the positive effect of Delta in a Call option that can be attributable to an increase in Call option spread. Also consider a negative impact of Delta for Put options. Other Greeks has a free interpretation related to its signs.

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[6]To not reject nullhipothesys the probability should be at least 0.5 or greater. (J.M Wooldridge)

Summing up, Bid – Ask spread of index options increases with Vega and decreases with Gamma while the remaining letters are uncorrelated for both options and have opposite but logical interpretation.

5. Conclusions

This paper exposed the effects of Greek letters in bid – ask spread prices of S&P´s options and how can be integrated Call and Put options incorporating statistical results. It was found that bid – ask spread for both index Call and Put options can be explained by Gamma and Vega that have equal effects in both cases. In both cases, Gamma and Vega was highly determinant parameters to explain the changes in options spread; it make sense considering that those parameters concerns about changes in the underlying asset, in this case, S&P´s index. another results implies that Call and Put option spread are significantly positive related to Vega in which case the spread is seem to be increased as changes in volatility.

The insignificance of Delta only make us conclude that changes in S&P´s prices does not affect that much options spreads and the significance becomes in the second derivative and the convexity of the options (Gamma). Finally, Theta does not indicate a clear prediction of how option spread can behave considering maturity date; as expressed before, this parameter has ambiguous results.

6. Appendix

TABLE A: Information scheme found in CBOE

Overview | Chart | News | Options | Portfolio

Name	Last	Change		
(SPX) S&P 500 INDEX	$1,446.79	16.43(1.15%)	Options Chain	Add to Portfolio

Chain Type: Calls and Puts | Chain Type: All | Expiration: Jan 2013 | View Chain

Calls								Puts				
Contract Name	Last Trade	Change	Bid	Ask	Volume	Interest	Strike Price	Contract Name	Last Trade	Change	Bid	Ask
SPX\13A19\100.0	0.00	0.00	1,343.80	1,346.30	0.00	0.00	100.00	SPX\13M19\100.0	0.00	0.00	0.00	0.05
SPX\13A19\200.0	0.00	0.00	1,243.90	1,246.40	0.00	0.00	200.00	SPX\13M19\200.0	0.00	0.00	0.00	0.05
SPX\13A19\300.0	0.00	0.00	1,143.90	1,146.40	0.00	0.00	300.00	SPX\13M19\300.0	0.00	0.00	0.00	0.05
SPX\13A19\400.0	0.00	0.00	1,044.00	1,046.20	0.00	0.00	400.00	SPX\13M19\400.0	0.00	0.00	0.00	0.05
SPX\13A19\500.0	925.23	66.73	944.00	946.50	200	2,700	500.00	SPX\13M19\500.0	0.05	0.00	0.00	0.05
SPX\13A19\525.0	0.00	0.00	914.60	925.90	0.00	0.00	525.00	SPX\13M19\525.0	0.10	0.00	0.05	0.05
SPX\13A19\550.0	0.00	0.00	894.10	896.30	0.00	0.00	550.00	SPX\13M19\550.0	0.15	-0.05	0.05	0.05
SPX\13A19\575.0	0.00	0.00	869.10	871.30	0.00	0.00	575.00	SPX\13M19\575.0	0.15	0.00	0.05	0.05
SPX\13A19\600.0	0.00	0.00	844.10	846.60	0.00	0.00	600.00	SPX\13M19\600.0	0.40	-0.35	0.05	0.05
SPX\13A19\625.0	0.00	0.00	819.10	821.30	0.00	0.00	625.00	SPX\13M19\625.0	0.05	-0.20	0.05	0.05
SPX\13A19\650.0	0.00	0.00	794.10	796.60	0.00	0.00	650.00	SPX\13M19\650.0	0.25	0.00	0.05	0.05
SPX\13A19\675.0	0.00	0.00	769.10	771.60	0.00	0.00	675.00	SPX\13M19\675.0	0.05	-0.05	0.05	0.05
SPX\13A19\700.0	0.00	0.00	744.10	746.60	0.00	0.00	700.00	SPX\13M19\700.0	0.15	0.10	0.05	0.05
SPX\13A19\725.0	0.00	0.00	714.70	725.90	0.00	0.00	725.00	SPX\13M19\725.0	0.05	0.00	0.05	0.05
SPX\13A19\750.0	0.00	0.00	694.20	696.60	0.00	0.00	750.00	SPX\13M19\750.0	0.05	0.00	0.05	0.05

TABLE B: Information found for each Call/Put

Company Data

Overview | Chart | News | Options | Portfolio

Name

(SPX Jan 19 2013 700.0 Call) CBOE S&P 500 OPEN/EURO

Price	
Last	0.00
Change	0.00
Change%	0.00
Bid	744.1
Ask	746.6

OHLC	
Open	0.00
High	0.00
Low	0.00
Prev Close	0.00
52 Wk High	0.00
52 Wk Low	0.00

Volume	
Volume	0.00
Option type	CALL
Strike Price	
Expiration Date	01/19/2013

Greeks	
Delta	1.00
Gamma	0.00
Vega	281.83
Theta	0.00

Postestimation testsfor OLSregression (Put Spread)

```
. hettest, iid

Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
         Ho: Constant variance
         Variables: fitted values of Spread

         chi2(1)      =     3.45
         Prob > chi2  =   0.0634
```

```
. sfrancia ee

                  Shapiro-Francia W' test for normal data

    Variable |   Obs        W'         V'         z       Prob>z
-------------+--------------------------------------------------
          ee |   171     0.98489    2.130      1.576     0.05753
```

Post estimation tests for OLS regression (Call Spread)

```
. hettest, iid

Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
         Ho: Constant variance
         Variables: fitted values of Spread

         chi2(1)      =     1.61
         Prob > chi2  =   0.2044
```

```
. ovtest

Ramsey RESET test using powers of the fitted values of Spread
       Ho:  model has no omitted variables
                  F(3, 157) =      2.43
                   Prob > F =      0.0673
```

```
. sfrancia ee

                    Shapiro-Francia W' test for normal data

    Variable |   Obs        W'          V'          z       Prob>z
-------------+--------------------------------------------------
          ee |   169     0.98438     2.180       1.622     0.05243
```

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